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PRICING IN INFORMATION ORCHESTRATORS AND MAXIMIZING STABLE NETWORKS

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Abstract

In innovation networks based on information exchange, an orchestrating actor, or hub, captures information from peripheral actors, promotes innovation and then distributes it to the network in the form of added value. This paper identifies the pricing options proposed by the orchestrating hub that would result in the network's stability and efficiency. Since all the companies in this ecosystem can be seen as rational agents, game theory is an appropriate framework for studying pricing as a mechanism to promote network stability. We analyze the equilibrium conditions in this context and conclude that the Nash equilibrium entails the network's stability. Our findings indicate that, in order to maximize the innovation power of the network, the agents should be charged a price proportional to the financial benefit obtained by the net innovation. This study fills relevant gaps in the literature on monopolistic orchestrated innovation and the pricing structures of network connections.

Keywords: Innovation Networks; Network Stability; Game Theory; Nash Equilibrium; Pricing.

Subject Classification: D85, L14, C70, C72.

1. Introduction

The efficiency of organizations is highly impacted by their connection to innovation networks. These networks may be established through the cooperation of independent companies (peripherical actors) or around a central actor (orchestrator) that manages the information flow and coordinates the generation of innovation for the other actors (Dhanaraj and Parkhe [2006]). Companies in a number of different sectors, such as retail, tourism, telecommunications and financial services, have opted for using external agencies provided with information which is shared to optimize value chain processes, reducing costs and increasing efficiency in decision-making.

One of the main objectives of the orchestrator is to promote its network's stability, that is, to keep the actors connected, avoiding that they migrate to other networks or disconnect. Since this kind of network is established through a client-supplier relationship, the price charged by the orchestrating company will be assessed and compared by the connected companies to the value effectively generated by the connection. Price is therefore a key coordination component in terms of the network's stability.

In this context, we try to answer the following questions: what is the relationship between pricing and the maintenance of networks' stability? What are the characteristics of a fair pricing structure?

The aforementioned environment can be seen as a game where both the peripheral actors and the orchestrating hub can be seen as economically rational agents who try to maximize their profits and where each company can adopt one of the two following strategies: to connect or not to connect to the network.

Hence, in order to answer the questions above, we propose a theoretical framework based on a simplified game in which an orchestrating hub manages information about companies which might connect to it and obtain financial gains from the innovation provided. Since every actor in the network can be interpreted as a rational agent, the conclusions on the effects of the different pricing models can be reached through the analysis of the conditions for a Nash equilibrium of the formulated game. The analytical interpretation of the inequalities encountered in the solution allows us to reach relevant conclusions on the effect of pricing on network coordination and stability. We also include an illustrative example which helps with the visualization and the interpretation of the conclusions reached analytically. Our results show that the network's stability is maximized when the price paid by the agents is proportional to the benefit obtained from participating in the network.

We contribute to the literature by, for the first time, associating pricing with stability and efficiency in different types of networks — complete, partial and empty — in the presence of externalities. Moreover, whilst studies in this area have considered a number of sectors, the financial segment has been less explored. We fill in this gap by illustrating our study with an example applicable to orchestrators such as credit rating agencies and credit bureaus who use the information of their own clients (e.g. banks, retail companies and tourism agencies), promote innovation through the storage, analysis and statistical modeling of the client's data, and pass the innovation back to their clients in the form of a summary of the credit risk of private individuals, allowing their clients to make better decisions.

Apart from the academic contributions, this paper has important practical implications. We focus on monopolistic markets where organizations acting as orchestrating hubs do not have a benchmark from peers to help in defining their prices. This scenario is usual in many circumstances and includes cases where a very large organization shares the market with small competitors. Our study provides a novel support to network managers when defining optimal prices for their services. In other words, our framework can guide orchestrators' managers who need to set prices for their services so that they guarantee the highest possible income without reducing the number of clients (i.e. keeping the network stable). This can be applied not only in the sector illustrated here but also in any business areas.

2. Literature Review

2.1. *Networks' structure and their stability*

Networks display different structures and have different typologies. They can be formal or informal, where the latter are those with free association or coexistence among participants and former are those in which participation is formally contracted. Networks can also be horizontal or vertical. Horizontal networks are cooperation networks in which the actors preserve their independence and vertical networks are hierarchical networks (Marcon and Moinet [2000]). Another variation of the structural form of networks is related to their density, which is measured by the ratio between the quantity of existing connections and the quantity of potential connections in the network when all participants are interconnected. The position of an actor in a network in relation to the other actors involved is understood as its centrality (Rowley [1997]).

Network stability, which has been investigated in a number of studies (e.g. Dutta and Mutuswami [1997]; Jackson and Van den Nouweland [2005]; Bartelings *et al.* [2017], can be understood as the capability of a network to reach a non-negative growth rate, even if it allows members to enter or exit the network (Dhanaraj and Parkhe [2006]).

The analyses on this issue have focused on very specific contexts, such as computer networks (Avrachenkov *et al.* [2015]; Delgado [2010]), telecommunications (Hong and Chun [2010]), small and medium enterprises (Nilsen and Gausdal [2017]), mechanical engineering industry (Landsperger and Spieth [2011]), R&D (König et al. [2012]) and supply networks (Ostrovsky [2008]). We expand the range of the sectors analyzed by focusing on the financial segment. Pricing has also been considered in the literature on networks (e.g. Bloch and Quérou [2013]; Fainmesser and Galeotti [2016]) but without a clear link with network stability, which is done in this paper.

2.2. *Game Theory, Nash Equilibrium and networks' stability*

Game Theory, just like the Nash Equilibrium, has been extensively used in the literature as the appropriate framework for understanding the formation and stability of networks because the strategies chosen by each actor influence the potential results of all actors (Avrachenkov et al. [2015]; Ozkan-Canbolat and Beraha [2016]; Anshelevich et al. [2008]; Goyal and Vega-Redondo [2005]; Bala and Goyal [2000]). The actors involved in networks can represent people or companies which, most of the times, choose to form or not to form their connections with the objective of maximizing their returns. From the Game Theory's standpoint, considering that the utility or result achieved by each player depends on the strategy used by each of them (Von Neumann and Morgenstern [1944]), the actors forming the network can be understood as rational agents who have two strategies in relation to each different actor in the network: to connect or not to connect.

When each agent achieves a better result in comparison to the result from the opposite decision (i.e. to connect to the network or not), none of them regret their decisions. If analyzed over time, such network would be stable because it is in the Nash equilibrium. If the members connected to the network do not regret doing it, it is expected that they will remain connected in the future. Likewise, the non-participating members would not decide to connect to the network. Given that network stability is defined as “non-negative growth rates”, the stationary state brought about by a Nash equilibrium (Nash [1951]) implies a stable network (Anshelevich et al. [2008]).

In fact, it is common the existence of many scenarios with Nash equilibria and, therefore, multiple possibilities of reaching network stability. As a consequence, it is common to focus on the relationship between the “best Nash equilibrium” and the “worst Nash equilibrium”, being the “best equilibrium” the one that incentivizes everyone’s participation. On the other hand, the “worst equilibrium” does not incentivize the formation of the network and provides the worst global result for the group.

3. Theoretical Framework

In the context of formal and horizontal innovation networks (Marcon and Moinet [2000]) constituted by rational agents (Mas-Colell *at al.* [1995]), this study considers different pricing models adopted by an orchestrating hub (Dhanaraj and Parkhe [2006]) as a network coordination instrument, and the relationship between these different models and the network’s stability (Dhanaraj and Parkhe [2006]) and density (Rowley [1997]), as well as the presence of externalities (Katz and Shapiro [1985]). We use Game Theory (Gibbons [1992]), in which the Nash equilibrium (Nash [1951]) implies network stability.

We investigate a particular type of innovation network characterized by low density (Rowley [1997]) and only one agent, the orchestrating hub, with high centrality (Freeman [1991]). In this type of network, companies with different volumes of information connect only to the orchestrating hub, who, in turn, creates added value with the set of information items received and make them available for all the actors connected to the network. Because it is based on free association and because the services provided are contracted, the network is defined as horizontal and formal, respectively (Marcon and Moinet [2000]). This type of structure is very common in the markets of credit-granting (Pagano and Jappelli [1993]), frauds, and debt collection.

All actors, including the orchestrating hub, are private companies, which makes the use of the economic theory of agents rationality even more feasible (Mas-Colell *et al.* [1995]). They are rational agents in the sense that they always aim at maximizing their utility functions (i.e. their financial results). Hence, it is reasonable to assume that the agents' primary motivation for connecting to the network is financial. Thus, the network's instrument of coordination would be the price stipulated by the orchestrating hub. Without loss of generality, other factors that could also interfere in the actors' decision (Meyer and Rowan [1977]) have been left out for simplification purposes.

The framework provided by Game Theory offers a valuable tool for assessing stability in networks, having been used many times towards this end (Anshelevich et al. [2008]; Papadimitriou [2001]). The environment can be understood as a game in which the actors choose between two strategies (to connect or not to connect) and benefit according to the result obtained by the network's innovation, measured against the price paid for the connection. Game Theory is appropriate to approach this problem, especially because it involves rational agents, which is more acceptable when we are dealing with companies than with individuals.

As defined by Nash [1951], an equilibrium occurs in a game when, once having chosen their strategies and assessed their results, none of the agents, or in this case, actors, regrets having chosen the strategy they picked, given the strategies picked by the other actors. Such equilibrium implies three possible situations: the non-existence of a network (empty network), the existence of the network with a partial quantity of participants (partial network), and the existence of the network with all possible participating actors (complete network). The last situation can be understood in the present study as "the best equilibrium" in terms of network stability. It is the scenario with maximum density for the context proposed.

Our study adds to the existing body of research on the relationships between the coordination structures and the potential levels of density and efficiency as well as the relationships between these scenarios and the presence of externalities. The central focus is on the pricing structure and the different implications it has on the density and on the presence of externalities. Different price structures are determined if a network is either *complete*, when it has maximum density, or *partial*, i.e., with density between zero and the maximum value.

4. The Model

4.1. *Model with price discrimination*

Let (S, f) be a game with *n* participants and S_i the set of strategies of player i. $S = S_1 X S_2 X \dots X S_n$ is the set of strategies which specifies all actions of a game. $f = (f_1(x), \dots, f_n(x))$ is the reward function. Let χ_{-i} be the set of strategies of all players, except for player i. When each player $i \in \{1, \dots, n\}$ selects its strategy x_i, the set of resulting strategies is $x = (x_1, \dots, x_n)$ and the player i has the reward function $f_i(x)$. A set of strategies $x^* \in S$ is a Nash equilibrium when $\forall i, x_i \in S_i, x_i \neq x_i^*: f_i(x_i^*, x_{-i}^*) \leq f_i(x_i, x_{-i}^*)$.

The reward function, $f_i(x)$, can be defined in terms of each peripheral actor *i*'s utility function *u* as:

$$u_i^0 = k_i c_1 + (1 - k_i) c_0 \qquad (1)$$

$$u_i^1 = k_r c_1 + (1 - k_r) c_0 + p_i \qquad (2)$$

where the superscripts 0 and 1 correspond to not joining or joining the network, respectively. k_i is the fraction of inhabitants known by peripheral actor i previously to them joining the network. k_r is the proportion of inhabitants known by the network, i.e., by the orchestrating hub. c_1 and c_0 are, respectively, the costs when the peripherical actor knows and does not know the inhabitant. Therefore, it is natural to assume that $c_1 < c_0$. p_i is the price charged by the orchestrating hub.

Given the Nash equilibrium conditions specified above and Eqs. (1) and (2), we have $u_i^1 \leq u_i^0, \forall i$, which results in:

$$(k_r - k_i)(c_0 - c_1) \geq p_i \tag{3}$$

Under this condition, it is possible to find a price per peripheral actor which maximizes the size of the network. Since $k_r \geq k_i$ and $c_0 > c_1$ by construction, then there is a $p_i \geq 0$ that allows for the equilibrium in any case. $(k_r - k_i)$ can be understood as the difference between the fraction of inhabitants known by the network and the fraction of inhabitants known by the company connected. The subtraction $(c_0 - c_1)$ represents the difference between the cost of making a decision for an unknown client and the cost of making a decision for a known client. Thus, $(c_0 - c_1)$ is the financial gain from the network's innovation for each client, whose decisions will now be made based on information from the network.

$(k_r - k_i)(c_0 - c_1)$ means the innovation gain appropriated by the network from the company connected in relation to the fraction of inhabitants whose decisions are made for a lower price due to the innovation network. This is the main conclusion of this work: the price established by the monopolistic orchestrating hub for each company connected to the network should be proportional to the financial gain from the innovation provided by the network for each peripheral actor.

Let R be the set of actors that have decided to connect to the network. The orchestrating hub's profit L consists of the sum of the prices paid by the peripheral actors net of the cost C incurred by the hub, i.e., $L = \sum_{i \in R} p_i - C$.

For a Nash equilibrium to be achieved in the game, no peripheral actor can regret the decision made, given the respective choices made by all of the other actors. Thus, the reward function of the strategy chosen by all actors must have a result which is equal to or lower than the result of the rejected strategy, since we are dealing with a problem of cost reduction.

4.2. *Model without price discrimination*

The model introduced in the previous section is based on the highly feasible principle that the orchestrating hub can define a specific price p_i for each peripheral actor i. Nonetheless, it is also important to adapt the model to situations where the orchestrating hub must charge all companies the same price p (e.g. due to a regulatory demand). In this case, $p_i = p, \forall i$.

4.2.1. *Empty network*

When no client is connected to the network, the number of actors in the network is given by an empty set $R = \{\emptyset\}$ and, consequently, the number of inhabitants known by the network is also given by an empty set. This means that the fraction of inhabitants known by the network k_r is zero.

In this case, all actors a_i have decided for the x_i^0 strategy, that is, not connecting to the network. In order to satisfy the Nash equilibrium condition in which none of the actors participate in the network, then $u_i^0 \leq u_i^1, \forall i$, i.e., the cost of isolation must be always lower than or equal to the cost of participating in a network in which the actor concerned would be the only participant.

In this context, using the previous notation, we have $k_i c_1 + (1 - k_i)c_0 \leq k_i c_1 + (1 - k_i)c_0 + p$, i.e., $p \geq 0$. Since $p \geq 0$ by construction, the Nash equilibrium would always occur. Thus, for any non-negative price stipulated by the orchestrating hub, the non-existence of the network is in itself a Nash equilibrium, as it has been shown in the literature (Bala and Goyal [2000]).

4.2.2. *Partial network*

In order to demonstrate the Nash equilibrium in partial or incomplete networks, we define a scenario with three actors, a_i, a_j, a_l, with three different levels of knowledge about the population, k_i, k_j, k_l, where $k_i < k_j < k_l$. Let us suppose a network has been formed by the actors a_i and a_j, where a_l opted for isolation, i.e., in the resulting network, the set of inhabitants known is the union of the inhabitants known by the actors a_i and a_j. A Nash equilibrium exists in the game when the conditions for equilibrium are satisfied for all peripheral actors.

First of all, let us analyze the condition for equilibrium of a_j. Since a_j opted for connecting, in order for this actor to be in equilibrium, its decision-making costs when participating in the network must be lower than or equal to its decision-making costs when in isolation, $u_i^1 \leq u_i^0$.

According to Inequality (3), a_j will be in Nash equilibrium whenever $p^* \leq \left(k_r - k_j\right)(c_0 - c_1)$, that is, as long as the single price stipulated by the orchestrating hub is lower than or equal to the financial gain from the innovation.

By analyzing the condition for equilibrium for a_i, we also come to $(k_r - k_i)(c_0 - c_1) \geq p$, by analogy. Substituting p with the price limit p^* paid by a_j, we have $(k_r - k_i)(c_0 - c_1) \geq \left(k_r - k_j\right)(c_0 - c_1)$. Since $k_i < k_j$, then $(k_r - k_i) > \left(k_r - k_j\right)$, thus, the price p^* which satisfies the equilibrium of a_j, also necessarily satisfies the equilibrium of a_i.

To complete the demonstration, we still have to show that a_l can be in equilibrium when in isolation from the network. For such, the isolation strategy adopted by a_l must have lower costs than the costs it would incur if it were participating in the network, that is, $u_l^0 \leq u_l^1$. We have that $u_l^0 = k_l c_1 + (1 - k_l)c_0$. To verify u_l^1, we need to consider the set of inhabitants known by the network if the actor a_l decided to participate in it (i.e. the summation of the inhabitants known by the three actors). In this case, the fraction (k_r') known by the network with the participation of a_l would be higher than or equal to the fraction of the resulting network k_r.

Developing the condition for equilibrium and considering the maximum price p^* that the orchestrating hub could stipulate so that a_j is in equilibrium, we have:

$$(k_r' - k_l) \leq \left(k_r - k_j\right)$$

The inequality indicates that, as long as the fraction of extra inhabitants known by the network that would be formed with the presence of a_l is smaller than or equal to the difference between the fraction of the existing network and the biggest participating actor in equilibrium, in this case, a_j, it will be beneficial for a_l to remain in isolation. This is explained by the fact that a_j is the biggest actor participating in the resulting network and it pays the maximum price p^* to be able to have a lower decision price for a fraction of the population. With this price determined by the orchestrating hub, a_j would only regret its isolation if it could observe that it could have a gain equal to or higher than the amount spared by the network formed due to its presence.

In practical terms, we can think of a credit bureau in which the biggest user can reduce the costs for 30% of the population, on top of those they already know. Naturally, the bureau would determine the price according to this fraction. The new user, then, would only be willing to pay the price in question if it could obtain a gain of the same nature in the new network in which it would participate.

4.2.3. *Complete network*

Making use of a similar context to the one formulated in the former section, we can study a network in which at least one actor has more information k about the population than the others, i.e., $\exists j \in R | k_j > k_i, \forall i \neq j$. If we set a specific price p^* such that $\left(k_r - k_j\right)(c_0 - c_1) = p^*$, since $k_j > k_i \Rightarrow (k_r - k_i)(c_0 - c_1) > p^*$, i.e., the same price that satisfies the equilibrium for the actor that has the most knowledge of the inhabitants of the region to participate in the network, it is easy to notice that $p*$ will necessarily satisfy the condition for equilibrium of all other peripheral actors which have less knowledge about the population.

As an example of this situation, imagine the price for which the biggest bank of a particular region would decide to participate in the network in equilibrium making use of a credit bureau. This same price would be beneficial for all other banks of the region as they could count on the same network innovation, though with a smaller client base. Such situation opens the door to the discussion on the presence of externalities in innovation networks. An externality occurs when the utility observed is different from the expected utility at the time actors join the network.

4.2.4. *Network effects without price discrimination*

To delve deeper into the issue of the presence of externalities in the pricing of the innovation network, we use the same scenario constructed in the section concerning partial networks, where we have three actors, a_i, a_j, a_l and the following order of the proportion of the population respectively known by those actors: $k_i < k_j < k_l$. Suppose that the orchestrating hub establishes a price p^* for the network, such that $\left(k_r - k_j\right)(c_0 - c_1) = p^*$. It has already been verified in the aforementioned section that the same p^* satisfying the equilibrium of participation for a_j will necessarily satisfy the equilibrium of participation for a_i. Let us see each case separately:

For actor a_j, since the condition for equilibrium is $\left(k_r - k_j\right)(c_0 - c_1) \geq p^*$, the actor finds Nash equilibrium in the participation by connecting to the network because the value p^* stipulated by the orchestrating hub is exactly the same as the economic value of the potential innovation appropriated by actor a_j.

For actor a_i, since $k_i < k_j \Rightarrow (k_r - k_i)(c_0 - c_1) > p^*$, this actor that knows less about the population is not in Nash equilibrium by participating in the network but it has a financial advantage in relation to actor a_j. This is explained by the fact that when a_i pays the same price paid by actor a_j, which has more knowledge about the population, it appropriates more innovation than a_j because its knowledge about the population is smaller and even so it can make use of the same complete potential of the network as a_j does.

In this case, we can calculate the value of the externality of actor a_i in relation to actor a_j. It is easy to verify that the price limit that would make actor a_i participate in the network would be $p = (k_r - k_i)(c_0 - c_1)$. However, the price paid was $p^* = \left(k_r - k_j\right)(c_0 - c_1) < p$. Calculating the difference between the two scenarios, we have the value of the positive externality of a_i in relation to a_j, which is called $e_{i,j}$, or externality of a_i in relation to a_j.

$$e_{i,j} = (k_r - k_i)(c_0 - c_1) - \left(k_r - k_j\right)(c_0 - c_1) = \left(k_j - k_i\right)(c_0 - c_1)$$

That is, for not working with discrimination of price for different actors, the orchestrating hub allowed actor a_i to have an economic advantage $\left(k_j - k_i\right)(c_0 - c_1)$ from the innovation whose value is higher than the economic value it paid, which was $p^* = \left(k_r - k_j\right)(c_0 - c_1)$.

Thus, when prices for different actors are not discriminated, the orchestrating hub allows the actors for whom the network has the highest added value to pay the same value as the other actors for whom the network has lower added value. The conclusions concerning externalities are valid both for partial and complete networks.

5. Simulations

5.1. *Market specification*

In order to illustrate the models introduced in Section 4, we simulate a market with 10,000 inhabitants and 20 companies (peripheral actors) where each company knows a set of inhabitants of the region but needs to make decisions concerning inhabitants it does not know. These decisions could, for example, refer to credit granting, fraud analysis, insurance policy issuance, or any other type of analysis for which having information about the inhabitants is necessary and useful.

A level of propensity to know inhabitants is assigned to each of the 20 companies by means of trials using adaptations of the exponential probability. Likewise, each of the inhabitants is assigned a level of propensity of being known. Both the companies and the inhabitants are assigned different levels in relation to one another in an attempt to reproduce distributions commonly encountered in the market (i.e. the companies have different sizes and the inhabitants have distinct levels of credit use). The distribution of inhabitants per company, as well as the proportion of inhabitants known by each company are shown in Fig. 1 - Panels A and B, respectively.

[Insert Fig. 1 here]

In this example, the orchestrating hub is a credit bureau and the 20 companies which can participate in the network are banks, financial companies and retailers that wish to grant credit to the inhabitants of the region. The orchestrating hub has a fixed cost of $3.00, no matter the quantity of actors connected to the network.

The companies may try to make credit granting decisions by themselves or connected to the network with different decision-making costs. When the companies try to make a decision in isolation, they have a cost of $10 for the clients they do not know and $1 for those they know. Company #17, for instance, has an average decision-making cost of $1(0.054)+$10(0.946)=$9.514.

5.2. *Scenario without price discrimination*

Assume that the orchestrating hub has stipulated a fixed price of $2.00 for any company interested to participate in the network and that the companies that decided to connect are those marked with a YES in the column "Connection" in Table 1 - Panel A.

In the last row of Panel A, we have the price stipulated by the orchestrating hub ($2.00). Just to the right of the price, we have the fixed cost incurred by the orchestrating hub, $3.00. Each row of the table represents a company, numbered from 1 to 20, with their respective actions and results. In the second column, we see that eight companies have connected to the network in this game. Thus, the profit enjoyed by the orchestrating hub is 8 x $2.00 - $3.00 = $13.00. In the third column, we have the proportion of the population known by each company, with the last row showing the proportion of the population known by the network. In this case, the union of the eight companies that have connected to the network resulted in the orchestrating hub knowing a proportion of 28.07%. The columns "Cost – Isolation" and "Cost – Participation" show the cost they would have or have had if they remained isolated or if they participated in the network, respectively. The column "Economies" represents in positive value the financial gain the company would have (or has had) for participating in the network in comparison with the financial gain associated with not participating. For example, company #2, which chooses not to participate in the network, has a decision-making cost of $9.82 since it knows only 1.98% of the population. If it had chosen to connect to the network, sharing, thus, the information it possessed and paying $2.00 per consultation, company #2 would have had an average decision-making cost of $9.39, which would have resulted in economies of $0.43 (=$9.82 - $9.39) in costs. Since it would have been preferable for the company to participate in the network, it regrets the strategy chosen. Therefore, the game formulated does not reach a Nash equilibrium.

The biggest company, #4, which can be interpreted as a large bank in the region, decides not to connect to the network. If it had chosen to connect, it would have had a disadvantage of $0.96 per consultation in comparison to its results in isolation. Thus, it does not regret its decision.

A Nash equilibrium can be verified in empty networks. For the data generated, as it is shown in Section 4.2.1, we have an equilibrium for not participating in the network for any non-negative price, once none of the companies regret being in isolation. To illustrate this possibility, an empty network was generated whose cost of connection is only one cent (see Table 1 - Panel B). Even so, none of the companies regret not having connected to the network, because they would be paying to use a network that contains only their own information.

The next simulation studies the case of the incomplete or partial networks in which a Nash equilibrium is achieved. In this case, all companies connected to the network obtain economies in their decision-making even considering that they paid the price stipulated by the orchestrating hub, while those which choose isolation also obtain a financial advantage for not paying the connection price. The simulation is run with the same 20 companies. The price stipulated by the orchestrating hub is $3.00. According to the results in Table 2 - Panel A, all companies have decided to connect paying the price defined, except for companies #4, #6 and #18, which are the biggest companies in terms of level of knowledge about the population. The resulting network, thus, is comprised of 17 companies and all of these achieve economies for using the network, which collectively knows 46.45% of the population. It is easy to notice that the connection to the network is more beneficial for some companies than for others. Company #13, for instance, which knows a very small proportion of the population, achieves average economies of $1.12 for using the network, even after paying the stipulated price. On the other hand, company #9 has economies of only $0.30, since alone it already knows 9.79% of the inhabitants. This is one more example of externality.

The three largest companies, #6, #18 and #4 choose not to connect to the network and do not regret their decisions either. If they had chosen to connect, the price charged by the orchestrating hub would make it unfeasible for the companies to have any economies with decision-making costs generated by the usage of the network. This suggests that the orchestrating hub should practice a lower price for these companies, once it wishes to maximize the quantity of companies connected and, consequently, its profit.

Still in the model without price discrimination, an interesting exercise is to calculate the highest price that the orchestrating hub could practice so that the network is complete and in equilibrium. We find that such maximum price is $2.57. This is the price limit that makes company #4 indifferent to participation, i.e., it does not regret having chosen to connect with the network[1].

In short, the simulation results in Table 1 - Panel B confirm that empty networks entail Nash equilibrium as shown in Section 4.2.1. We also see that the higher the proportion k_i of the population known by company *i* the less it benefits from joining the network (see Table 1 - Panel A and Table 2 - Panel A). Consequently, the size of the population known *a priori* by companies

[1] The detailed results are not reported here due to space constraints but they are available upon request.

is a vital information that should be considered by orchestrators when defining their prices. This corroborates the models presented in Section 4.2.

5.3. *Scenario with price discrimination*

In the previous simulations, the orchestrating hub stipulates one single price for all of the 20 companies (peripheral actors). Now we discuss the simulation of the model presented in Section 4.1. In this model, the orchestrating hub raises the prices for each company up to the limit that would make them not regret having connected to the complete network. Naturally, then, the orchestrating hub appropriates itself of all the economies generated by the network and leaves all companies with zero economies. In doing so, the orchestrating hub has maximum profit, as shown in Table 2 - Panel B.

Based on these results, we conclude that the profit of the orchestrating hub with price discrimination is much higher than in the simulation without price discrimination: $98.16 *versus* $48.40. We also note that the price defined for each company is inversely proportional to the population known by it. The prices can vary from $2.57 for company #4 to $5.73 for companies #13 and #14, which know a very small proportion of the population.

Therefore, our findings in this section indicate that price discrimination leads to higher profits compared to situations where prices are the same for all actors and, as found in the previous section and in accordance with the models in Section 4, the size of the population known *a priori* by companies should be taken into account in the price definition.

Table 1. Results of a partial and an empty network.

Company	**Panel A – Partial network with a price of $2.00**						**Panel B – Empty network with a price of $0.01**					
	Connection	**Proportion known**	**Cost - Isolation**	**Cost - Participation**	**Economies**	**Regrets**	**Connection**	**Proportion known**	**Cost - Isolation**	**Cost - Participation**	**Economies**	**Regrets**
1	YES	5.56%	$ 9.50	$ 9.47	$ 0.03	NO	NO	5.56%	$ 9.50	$ 9.51	($ 0.01)	NO
2	NO	1.98%	$ 9.82	$ 9.39	$ 0.43	YES	NO	1.98%	$ 9.82	$ 9.83	($ 0.01)	NO
3	YES	4.93%	$ 9.56	$ 9.47	$ 0.08	NO	NO	4.93%	$ 9.56	$ 9.57	($ 0.01)	NO
4	NO	35.76%	$ 6.78	$ 7.74	($ 0.96)	NO	NO	35.76%	$ 6.78	$ 6.79	($ 0.01)	NO
5	NO	2.34%	$ 9.79	$ 9.36	$ 0.43	YES	NO	2.34%	$ 9.79	$ 9.80	($ 0.01)	NO
6	NO	20.68%	$ 8.14	$ 8.52	($ 0.38)	NO	NO	20.68%	$ 8.14	$ 8.15	($ 0.01)	NO
7	NO	4.69%	$ 9.58	$ 9.24	$ 0.34	YES	NO	4.69%	$ 9.58	$ 9.59	($ 0.01)	NO
8	YES	3.74%	$ 9.66	$ 9.47	$ 0.19	NO	NO	3.74%	$ 9.66	$ 9.67	($ 0.01)	NO
9	NO	9.79%	$ 9.12	$ 8.98	$ 0.14	YES	NO	9.79%	$ 9.12	$ 9.13	($ 0.01)	NO
10	NO	6.76%	$ 9.39	$ 9.17	$ 0.22	YES	NO	6.76%	$ 9.39	$ 9.40	($ 0.01)	NO
11	NO	9.52%	$ 9.14	$ 9.04	$ 0.10	YES	NO	9.52%	$ 9.14	$ 9.15	($ 0.01)	NO
12	YES	2.31%	$ 9.79	$ 9.47	$ 0.32	NO	NO	2.31%	$ 9.79	$ 9.80	($ 0.01)	NO
13	NO	0.65%	$ 9.94	$ 9.44	$ 0.50	YES	NO	0.65%	$ 9.94	$ 9.95	($ 0.01)	NO
14	YES	0.67%	$ 9.94	$ 9.47	$ 0.47	NO	NO	0.67%	$ 9.94	$ 9.95	($ 0.01)	NO
15	YES	2.51%	$ 9.77	$ 9.47	$ 0.30	NO	NO	2.51%	$ 9.77	$ 9.78	($ 0.01)	NO
16	NO	3.24%	$ 9.71	$ 9.32	$ 0.39	YES	NO	3.24%	$ 9.71	$ 9.72	($ 0.01)	NO
17	NO	5.4%	$ 9.51	$ 9.24	$ 0.28	YES	NO	5.4%	$ 9.51	$ 9.52	($ 0.01)	NO
18	NO	25.01%	$ 7.75	$ 8.32	($ 0.57)	NO	NO	25.01%	$ 7.75	$ 7.76	($ 0.01)	NO
19	YES	9.55%	$ 9.14	$ 9.47	($ 0.33)	YES	NO	9.55%	$ 9.14	$ 9.15	($ 0.01)	NO
20	YES	7.93%	$ 9.29	$ 9.47	($ 0.19)	YES	NO	7.93%	$ 9.29	$ 9.30	($ 0.01)	NO
Network	**8 companies**	**28.07%**	**Price: $2.00**	**Cost-network: $3.00**	**Profit:**	**$13.00**	**0 companies**	**0.00%**	**Price: $0.01**	**Cost-network: $3.00**	**Profit:**	**($3.00)**

Table 2. Results of networks in equilibrium.

Company	**Panel A – Partial network with a fixed price of $3.00**						**Panel B – Complete network with price discrimination**					
	Connection	**Proportion known**	**Cost - Isolation**	**Cost - Participation**	**Economies**	**Regrets**	**Connection**	**Proportion known**	**Cost - Isolation**	**Cost - Participation**	**Economies**	**Regrets**
1	YES	5.56%	$ 9.50	$ 8.82	$ 0.68	NO	YES	5.56%	$ 9.50	$ 9.50	$ 0	NO
2	YES	1.98%	$ 9.82	$ 8.82	$ 1.00	NO	YES	1.98%	$ 9.82	$ 9.82	$ 0	NO
3	YES	4.93%	$ 9.56	$ 8.82	$ 0.74	NO	YES	4.93%	$ 9.56	$ 9.56	$ 0	NO
4	NO	35.76%	$ 6.78	$ 7.85	($ 1.07)	NO	YES	35.76%	$ 6.78	$ 6.78	$ 0	NO
5	YES	2.34%	$ 9.79	$ 8.82	$ 0.97	NO	YES	2.34%	$ 9.79	$ 9.79	$ 0	NO
6	NO	20.68%	$ 8.14	$ 8.27	($ 0.13)	NO	YES	20.68%	$ 8.14	$ 8.14	$ 0	NO
7	YES	4.69%	$ 9.58	$ 8.82	$ 0.76	NO	YES	4.69%	$ 9.58	$ 9.58	$ 0	NO
8	YES	3.74%	$ 9.66	$ 8.82	$ 0.84	NO	YES	3.74%	$ 9.66	$ 9.66	$ 0	NO
9	YES	9.79%	$ 9.12	$ 8.82	$ 0.30	NO	YES	9.79%	$ 9.12	$ 9.12	$ 0	NO
10	YES	6.76%	$ 9.39	$ 8.82	$ 0.57	NO	YES	6.76%	$ 9.39	$ 9.39	$ 0	NO
11	YES	9.52%	$ 9.14	$ 8.82	$ 0.32	NO	YES	9.52%	$ 9.14	$ 9.14	$ 0	NO
12	YES	2.31%	$ 9.79	$ 8.82	$ 0.97	NO	YES	2.31%	$ 9.79	$ 9.79	$ 0	NO
13	YES	0.65%	$ 9.94	$ 8.82	$ 1.12	NO	YES	0.65%	$ 9.94	$ 9.99	$ 0	NO
14	YES	0.67%	$ 9.94	$ 8.82	$ 1.12	NO	YES	0.67%	$ 9.94	$ 9.94	$ 0	NO
15	YES	2.51%	$ 9.77	$ 8.82	$ 0.95	NO	YES	2.51%	$ 9.77	$ 9.77	$ 0	NO
16	YES	3.24%	$ 9.71	$ 8.82	$ 0.89	NO	YES	3.24%	$ 9.71	$ 9.71	$ 0	NO
17	YES	5.4%	$ 9.51	$ 8.82	$ 0.69	NO	YES	5.4%	$ 9.51	$ 9.51	$ 0	NO
18	NO	25.01%	$ 7.75	$ 8.19	($ 0.44)	NO	YES	25.01%	$ 7.75	$ 7.75	$ 0	NO
19	YES	9.55%	$ 9.14	$ 8.82	$ 0.32	NO	YES	9.55%	$ 9.14	$ 9.14	$ 0	NO
20	YES	7.93%	$ 9.29	$ 8.82	$ 0.47	NO	YES	7.93%	$ 9.29	$ 9.29	$ 0	NO
Network	**17 companies**	**46.45%**	**Price: $3.00**	**Cost-network: $3.00**	**Profit:**	**$48.40**	**20 companies**	**64.35%**	**Price: Variable**	**Cost-network: $3.00**	**Profit:**	**$98.16**

5.4. *Scenario with price negotiation*

In the previous discussions, we assumed that prices are fixed by the orchestrating hub and accepted by actors without any negotiation. However, in practice, it is very important to assess network stability when agents are allowed to negotiate prices. In this section, company #4 (the one with most information about the consumer market) is allowed to negotiate prices with the credit bureau. In these circumstances, we initially analyze a network in equilibrium made up of all companies, except for company #4.

The network formed with 19 participants knows 58.02% of the inhabitants of the region. If it is allowed that company #4 negotiates the price of its participation with the orchestrating hub and the orchestrating hub aggregates company #4 to the network, it increases the proportion of inhabitants known from 58.02% to 64.35%. Since the price each company is willing to pay is proportional to the difference between the proportion of inhabitants known by the network and the proportion of inhabitants known by each company, the orchestrating hub knows it could raise the price charged to all of the other companies in case company #4 participated in the network. It is beneficial for the orchestrating hub, for example, to let company #4 join the network for free, because doing so would increase its profit from $84.76 to $95.59. This result (not shown in Table 3) is obtained by adding up the price paid by each of the 19 companies in Table 3 - Panel A minus the fixed cost of $3.00 incurred by the network.

Table 3. Networks with price negotiation.

Company	**Panel A – All companies connecting except for company #4**						**Panel B – All companies connecting (including company #4)**					
	Proportion known	**Cost - Isolation**	**Cost - Participation**	**Prices**	**Economies**	**Regrets**	**Proportion known**	**Cost - Isolation**	**Cost - Participation**	**Prices**	**Economies**	**Regrets**
1	5.56%	$ 9.50	$ 9.50	$ 4.72	0	NO	5.56%	$ 9.50	$ 9.50	$ 5.29	0	NO
2	1.98%	$ 9.82	$ 9.82	$ 5.04	0	NO	1.98%	$ 9.82	$ 9.82	$ 5.61	0	NO
3	4.93%	$ 9.56	$ 9.56	$ 4.78	0	NO	4.93%	$ 9.56	$ 9.56	$ 5.35	0	NO
4	35.76%						35.76%	$ 6.78	($ 7.26)	($ 10.83)	14.04	NO
5	2.34%	$ 9.79	$ 9.79	$ 5.01	0	NO	2.34%	$ 9.79	$ 9.79	$ 5.58	0	NO
6	20.68%	$ 8.14	$ 8.14	$ 3.36	0	NO	20.68%	$ 8.14	$ 8.14	$ 3.93	0	NO
7	4.69%	$ 9.58	$ 9.58	$ 4.80	0	NO	4.69%	$ 9.58	$ 9.58	$ 5.37	0	NO
8	3.74%	$ 9.66	$ 9.66	$ 4.89	0	NO	3.74%	$ 9.66	$ 9.66	$ 5.45	0	NO
9	9.79%	$ 9.12	$ 9.12	$ 4.34	0	NO	9.79%	$ 9.12	$ 9.12	$ 4.91	0	NO
10	6.76%	$ 9.39	$ 9.39	$ 4.61	0	NO	6.76%	$ 9.39	$ 9.39	$ 5.18	0	NO
11	9.52%	$ 9.14	$ 9.14	$ 4.37	0	NO	9.52%	$ 9.14	$ 9.14	$ 4.93	0	NO
12	2.31%	$ 9.79	$ 9.79	$ 5.01	0	NO	2.31%	$ 9.79	$ 9.79	$ 5.58	0	NO
13	0.65%	$ 9.94	$ 9.99	$ 5.16	0	NO	0.65%	$ 9.94	$ 9.94	$ 5.73	0	NO
14	0.67%	$ 9.94	$ 9.94	$ 5.16	0	NO	0.67%	$ 9.94	$ 9.94	$ 5.73	0	NO
15	2.51%	$ 9.77	$ 9.77	$ 5.00	0	NO	2.51%	$ 9.77	$ 9.77	$ 5.57	0	NO
16	3.24%	$ 9.71	$ 9.71	$ 4.93	0	NO	3.24%	$ 9.71	$ 9.71	$ 5.50	0	NO
17	5.4%	$ 9.51	$ 9.51	$ 4.74	0	NO	5.4%	$ 9.51	$ 9.51	$ 5.31	0	NO
18	25.01%	$ 7.75	$ 7.75	$ 2.97	0	NO	25.01%	$ 7.75	$ 7.75	$ 3.54	0	NO
19	9.55%	$ 9.14	$ 9.14	$ 4.36	0	NO	9.55%	$ 9.14	$ 9.14	$ 4.93	0	NO
20	7.93%	$ 9.29	$ 9.29	$ 4.51	0	NO	7.93%	$ 9.29	$ 9.29	$ 5.08	0	NO
Network	**58.02%**	**Price: Variable**	**Cost-network: $3.00**		**Profit:**	**$84.76**	**64.35%**	**Price: Variable**	**Cost-network: $3.00**		**Profit:**	**$84.76**

Following this line of reasoning, if the credit bureau wishes to increase or keep its profit, it could pay company #4 to join the network up to the limit that would make its profit be higher than or equal to $84.76. That is, the credit bureau would be willing to pay $10.83 to company #4. The results are shown in Table 3 - Panel B and indicate equilibrium as no company regrets participating in the network.

It is worth noting that the value of each company to the network depends not only on the proportion of the population the company knows, but also on the proportion of clients known by the company but unknown by the network. In our example, company #4 knows by itself 35.76% of the population, but what it adds to the network in terms of information is 6.33%, which is the proportion of inhabitants that the network in Table 3 - Panel B has beyond that which the network in Table 3 - Panel A has.

These results reveal that an orchestrator should observe the ratio of the population known by each actor and by the network when setting the price for its service. Hence, companies would be encouraged to join the network because the price stipulated would bring about financial advantages to them. As a consequence, the network and, in particular, the orchestrator hub would also benefit from the increasing number of members.

In sum, all the scenarios considered above support the conclusion that pricing is intimately associated with the network's stability, density and profitability in different types of networks.

6. Conclusions

We use Game Theory to analyze the relationship between the pricing structures proposed by the orchestrating hub and the network's stability and efficiency. This is novel in the literature and has yielded unprecedented conclusions that will be useful for decision-makers in different companies, such as bureaus for credit information, and fraud and insurance claims.

Our findings can be summarized as follows. First, the price structure defined *a priori* by the orchestrating hub is related to the network's stability. Different price structures have direct impact on the trend of permanence, increase or deterioration of the network. Second, the pricing structure proposed is related to the network's efficiency and density. Structures with or without price discrimination for the peripheral actors and their respective prices can in themselves imply empty, partial or complete networks. Third, pricing structures without price discrimination for actors of different sizes imply the presence of externalities in the network. Last but not least, the pricing structure which maximizes the network's density and eliminates externalities is that in which the price is proportional to the gains from the innovation appropriated by the peripheral actors.

From the academic point of view, where only cost allocation structures are studied, such framework contributes to filling in an existing gap in the study of horizontal, formal networks endowed with orchestrating hubs. In the scenario in which the actors are companies and the connections are established for contracting services, the price charged has proved to be an important tool in promoting the network's stability, which is described as one of the three main objectives to be promoted by the orchestrating hub (Dhanaraj and Parkhe [2006]).

This work, however, has some limitations, although none of them depreciates the conclusions reached for the proposed scenario. First and foremost, the market studied here is a monopolistic one. The presence of a competing orchestrating hub would change the study and is beyond our scope. This is, therefore, a suggestion for further studies in this area.

For simplification, we assume that costs incurred by the orchestrating hub are fixed, no matter the number of actors connected to the network. It would be reasonable to assume that costs depend on how many actors are connected to the network. Nonetheless, using variable costs in this study would have made it more complex without changing its conclusions as far as the decisions of the peripheral actors are concerned. This issue remains as an idea for extending the models developed here.

The assumption of rationality of companies and individuals that may influence stability can also be seen as a limitation of this study. In order to apply Game Theory, the starting point has to be the agents' rationality and, in this case, the price would be the determining factor in the context proposed in our analyses. It is known that other factors may interfere in decision making but such factors are not in the scope of the present work and their inclusion is left for future research.

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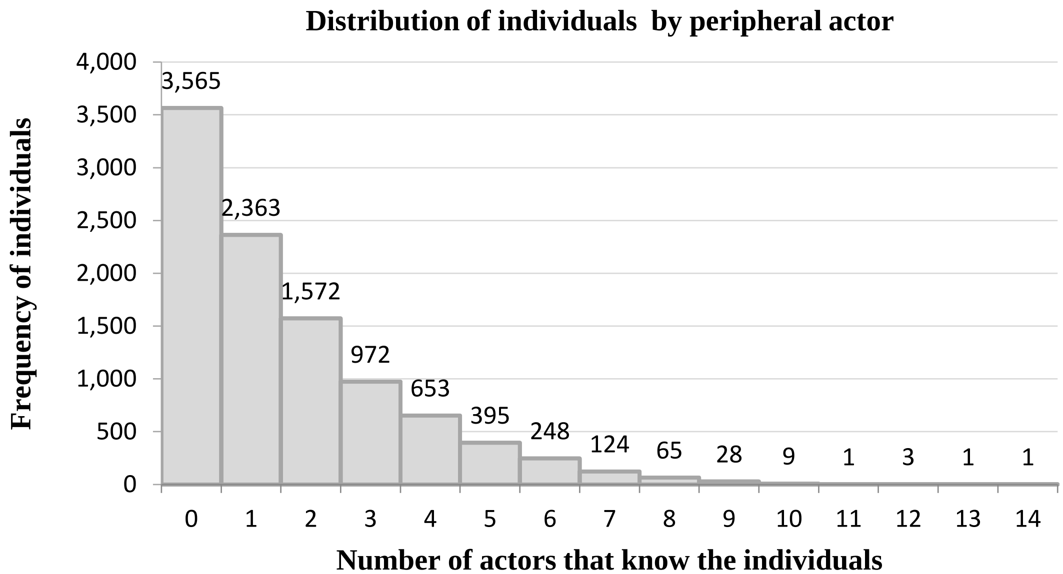
Panel A – Distribution of individuals by peripheral actor



Panel B – Proportion of the inhabitants known by each peripheral actor

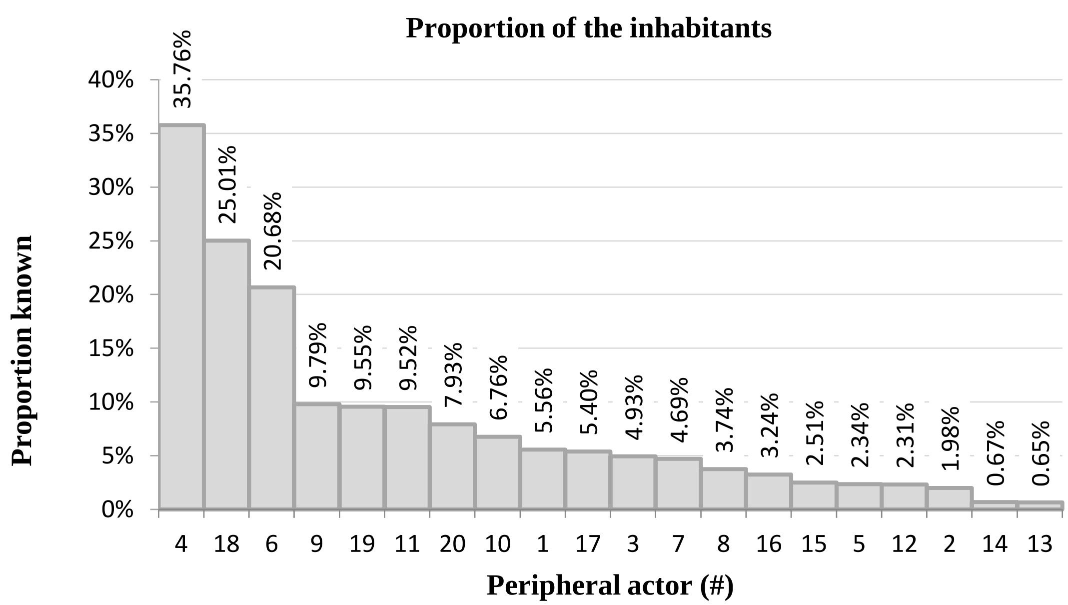


Fig 1. Distribution of simulated data.